

THEORETICAL AND REAL WORKING CYCLE OF FOUR STROKE PISTON ENGINE

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Abstract

The most important problems concerning analyses of generalized computational thermodynamical working cycle of the four-stroke combustion engine are result of the identification of real indicating diagram. The analysis of working cycle of the four-stroke combustion engine is realized from point of view heat efficiency, the average theoretical cycle pressure, as well as of the influence of the organization working cycle on the value of the maximum pressure in the cylinder of a combustion engine. A novel mathematical model reflecting elementary processes occurring in the cylinder of a combustion engine and connected with it intake and exhaust systems of engine were proposed.

Thermodynamical working cycles of four-stroke piston engines, theoretical working cycle of a four-stroke piston engine, thermal efficiency of the theoretical thermodynamic generalised engine work cycle, mean theoretical pressure of the generalised thermodynamic work cycle of a combustion engine, maximum pressure of the theoretical thermodynamic work cycle of a combustion engine, comparative working cycles of piston engines, real working cycle of four-stroke piston engines are presented In het paper. The methods of analyses and calculations referring these engines work cycles were presented. These method show advisability and needs of using them to theoretical calculations and analyses of real work cycles. Generalized engine work cycle was proposed.

Keywords: *combustion engine, thermodynamics, working cycle, cylinder pressure*

1. Introduction

New methods of the analysis working cycles of four-stroke piston engines enable assessment of design parameters of a combustion engine. Most important problems concerning analyses of

generalized computational thermodynamical working cycle of the four-stroke combustion engine are result of the identification of real indicating diagram. The analysis of working cycle of the four-stroke combustion engine is realized from point of view heat efficiency, the average theoretical cycle pressure, as well as of the influence of the organization working cycle on the value of the maximum pressure in the cylinder of a combustion engine. A novel mathematical model reflecting elementary processes occurring in the cylinder of a combustion engine and connected with it intake and exhaust systems of engine are proposed.

The thermodynamic work cycle of a four-stroke piston engine presented in the p-V or T-s system is a circular cycle proceeding clockwise, consisting of quasistatic reversible balance transformations realised by the working medium being an ideal, semi-ideal, or real gas, during which the delivered heat (emitted during the combustion process) is converted into mechanical work.

Let us consider the conditions for obtaining the largest work of any engine work cycle presented in the p-v coordinate system and realised by a unit quantity of the working medium.

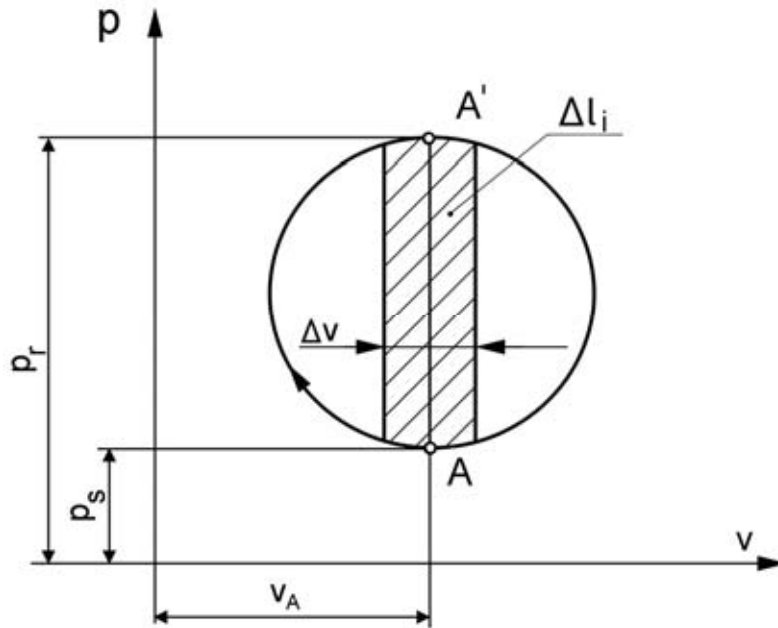


Fig. 1. A thermodynamical working cycle of a combustion engine in the p-v system

The value of the elementary indicated work Δl_i is:

$$\begin{aligned} \Delta l_i &= p_r \Delta v - p_s \Delta v = (p_r - p_s) \Delta v = \left(\frac{RT_r}{v_A} - \frac{RT_s}{v_A} \right) \Delta v = \\ &= R(T_r - T_s) \frac{\Delta v}{v_A} = \left(\frac{\bar{R}}{\mu_r} \frac{T_r}{v_A} - \frac{\bar{R}}{\mu_s} \frac{T_s}{v_A} \right) \Delta v = \bar{R} \left(\frac{T_r}{\mu_r} - \frac{T_s}{\mu_s} \right) \frac{\Delta v}{v_A} \end{aligned} \quad (1)$$

where: p_s – pressure of the compression process; p_r – pressure of the expansion process; v – specific volume of the working medium; R – individual gas constant; \bar{R} – universal gas constant; T_s and T_r – temperature of the working medium during the compression and expansion process, respectively, μ_s and μ_r – molar mass of the working medium during the compression and expansion process, respectively.

From relationship (1) it follows that the largest possible value of work Δl_i is obtained when:

- there is a large difference between the expansion and compression process pressure,

- there is a large value of the individual gas constant. Because $R = c_p - c_v = c_v(\kappa - 1)$, then the working medium should be characterized by a large specific heat c_v and a large value of the exponent of the adiabatic curve $\kappa = c_p/c_v$,
- there is a large difference between the expansion and compression process temperature. That condition is simultaneously the condition of high thermal efficiency of the work cycle,
- there is a possibly small value of the specific volume of the working medium v_A , which indicates that we should construct engines with high pressures existing in the cylinder, because then the values of the specific volume of the working medium are smaller.

From analysis of the last two expressions of equation (1) it follows that the value of the elementary indicated work Δl_i depends also on the molar mass of the fresh air-fuel mixture μ_s and of the combustion products μ_r .

In the field of piston engines, both during development of new designs and during improvement of existing ones, their thermodynamic work cycles are used, which are divided into: theoretical, comparative, and effective.

2. Theoretical working cycle of a four-stroke piston engine

Theoretical engine work cycles are standards used in the thermal engineering, which are used for presentation and analysis of the ideal course of the conversion of thermal energy into mechanical energy. The thermodynamic medium of those cycles is the ideal gas. They make it possible to compare the effective changes occurring in an engine with the theoretical changes according to which the engine should operate. They permit the drawing of conclusions concerning the theoretical efficiency and the mean theoretical pressure and the maximum pressure of the cycle, as well as the determination of the maximum values of parameters and indicators of engine operation, at the assumed characteristic quantities.

Theoretical thermodynamic engine work cycles are made using the following assumptions:

- the mass of the working medium in the cylinder is constant during the cycle,
- the compression and expansion processes proceed in isentropic manner,
- the heat delivered to the working medium, and the heat carried away from it, may be isochoric, isobaric, or isochoric and isobaric,
- the specific heats at constant volume and at constant pressure of the thermodynamic medium being subject to the transformation are constant,
- the thermodynamic transformations of the engine work cycle are reversible and occur infinitely slowly (quasistatically), therefore the medium velocities are equal to zero and flow losses do not occur.

The theoretical thermodynamic engine work cycle of the highest efficiency is the Carnot cycle, consisting of two isentropes (compression and expansion) and two isotherms (delivering and carrying away of heat). That cycle, however, has no application as a theoretical work cycle of a piston engine, because the compression transformation that would simultaneously connect the isotherms e.g. 290 K and 2900 K without exceeding the value p_{max} is impossible.

Let us consider a generalised theoretical thermodynamic work cycle of a four-stroke piston engine. The relationships derived below, describing its theoretical efficiency, mean theoretical pressure, or the herein formulated conclusions are right for all work cycles used in the theory of internal combustion piston engines, such as: Otto, Joule, Diesel, and Sabathe cycles. The generalised work cycle in p-v and T-s coordinates, presented in Fig. 2, is realised by 1 kg of the working medium.

3. Thermal efficiency of the theoretical thermodynamic generalised engine work cycle

The quantity of heat delivered to the cycle is:

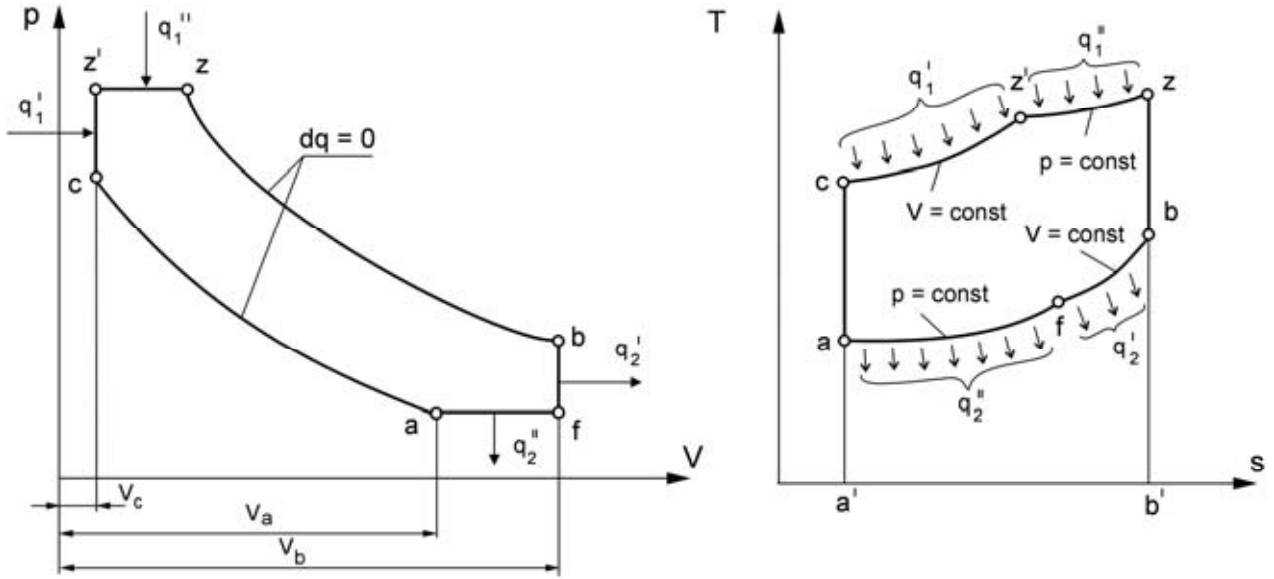


Fig. 2. A theoretical generalised work cycle of a four-stroke piston engine

$$q_1 = q_1' + q_1'' = c_v(T_{z'} - T_c) + c_p(T_z - T_{z'}), \quad (2)$$

where: c_v and c_p – specific mass heats of the working medium at constant volume and constant pressure, respectively; T_c , $T_{z'}$ and T_z – temperatures in respective points of the cycle, in the T-s coordinate system: $q_1 \sim s_{a'acz'zbb'}$.

The absolute quantity of the heat carried away from the cycle is:

$$|q_2| = c_v(T_b - T_f) + c_p(T_f - T_a), \quad (3)$$

where: T_b , T_f and T_a – cycle temperature in the points b, f, and a, in T-s coordinates, $|q_2| \sim s_{a'afbb'}$.

Therefore the thermal efficiency of the cycle is:

$$\eta_t = 1 - \frac{|q_2|}{q_1} = 1 - \frac{c_v[(T_b - T_f) + \kappa(T_f - T_a)]}{c_v[(T_{z'} - T_c) + \kappa(T_z - T_{z'})]}, \quad (4)$$

where: $\kappa = \frac{c_p}{c_v}$ - exponent of the adiabatic curve of the working medium.

Let us adopt the following designations:

$\varepsilon = \frac{V_a}{V_c}$ – compression ratio; $\lambda_p = \frac{p_z}{p_c}$ – pressure increase ratio during the delivery of heat to the cycle at $V = \text{const}$; $\rho = \frac{V_z}{V_c}$ – expansion ratio during the delivery of heat to the cycle at $p = \text{const}$;

$\delta = \frac{V_b}{V_z}$ – ratio of the consecutive expansion; $\rho' = \frac{V_b}{V_a} = \frac{V_f}{V_a}$ – momentary compression ratio

during the giving up of heat to the cool source, at $p = \text{const}$; $\frac{\varepsilon}{\delta} = \frac{\rho}{\rho'}$.

Utilising the equations of characteristic transformations of the cycle, and expressing the temperatures occurring in equation (4) using the introduced designations and the temperature at the beginning of compression T_a , after transformations we obtain:

$$\eta_t = 1 - \frac{\lambda_p \rho \left(\frac{\varepsilon}{\delta} \right)^{\kappa-1} + \frac{\delta \rho}{\varepsilon} (\kappa-1) - \kappa}{\varepsilon^{\kappa-1} \left[(\lambda_p - 1) + \kappa \cdot \lambda_p (\rho - 1) \right]} \quad (5)$$

From that relationship it follows that the theoretical thermal efficiency of the generalised thermodynamic engine work cycle depends on the engine design parameters expressed by the quantity ε , the quality of fuel fed to the engine expressed by the value of the exponent of the adiabatic curve κ , and the organization of the combustion process defined by the quantities λ_p , ρ and δ . Formula (5) is true for all work cycles used in the theory of thermodynamic engines. Let the following examples justify that statement.

Otto cycle. For that cycle we have: $\varepsilon = \delta$, $\rho = 1$. Substituting those values to formula (4) we obtain:

$$\eta_t = 1 - \frac{\lambda_p + \kappa - 1 - \kappa}{\varepsilon^{\kappa-1} (\lambda_p - 1)} = 1 - \frac{1}{\varepsilon^{\kappa-1}} \quad (6)$$

Sabathe cycle. For that cycle we have: $V_a = V_b = V_f$, i.e. $\varepsilon = \delta \rho$. Substituting the above relationships to (5), we obtain:

$$\eta_t = 1 - \frac{\lambda_p \rho \varepsilon^{\kappa-1} + \kappa - 1 - \kappa}{\varepsilon^{\kappa-1} \left[(\lambda_p - 1) + \kappa \cdot \lambda_p (\rho - 1) \right]} = 1 - \frac{\lambda_p \rho^{\kappa} - 1}{\varepsilon^{\kappa-1} \left[(\lambda_p - 1) + \kappa \cdot \lambda_p (\rho - 1) \right]} \quad (7)$$

4. Mean theoretical pressure of the generalised thermodynamic work cycle of a combustion engine

Because the work of the theoretical thermodynamic generalised work cycle of a combustion engine is $l_t = q_1 \cdot \eta_t$, then according to the definition of the mean theoretical pressure we have:

$$p_t = \frac{L_t}{V_s} = \frac{l_t}{v_s} \quad (8)$$

$$q_1 = c_v (T_z - T_c) + c_p (T_z - T_z') = c_v T_a \varepsilon^{\kappa-1} \left[\lambda_p - 1 + \kappa \lambda_p (\rho - 1) \right] \quad (9)$$

$$V_s = V_f - V_c = V_c \left(\frac{V_f}{V_c} - 1 \right) = V_c \left(\frac{V_a}{V_c} \cdot \frac{V_f}{V_a} - 1 \right) = \frac{V_a}{\varepsilon} (\varepsilon \cdot \rho' - 1) \quad (10)$$

Substituting the above relationships to (8) and utilising the relationships: $c_v = \frac{R}{\kappa - 1}$ and $p_a = \frac{RT_a}{V_a}$, after transformations we finally obtain:

$$p_t = c_v \frac{T_a \varepsilon^\kappa}{V_a (\rho' \cdot \varepsilon - 1)} [\lambda_p - 1 + \kappa \lambda_p (\rho - 1)] \eta_t = \frac{p_a \varepsilon^\kappa}{(\kappa - 1)(\varepsilon \cdot \rho' - 1)} [\lambda_p - 1 + \lambda_p (\rho - 1)] \cdot \eta_t. \quad (11)$$

For Sabathe cycle $\rho' = 1$, therefore we obtain:

$$p_t = \frac{p_a \varepsilon^\kappa}{(\kappa - 1)(\varepsilon - 1)} [\lambda_p - 1 + \lambda_p (\rho - 1)] \eta_t. \quad (12)$$

From that formula it follows that the effective way to increase the value of mean theoretical pressure, and therefore also the engine power, is the increase of the value of pressure of the beginning of compression p_a . One of the ways to increase of the value p_a is the use of combustion engine supercharging.

4. Maximum pressure of the theoretical thermodynamic work cycle of a combustion engine

The value of maximum pressure of the theoretical cycle of an internal combustion engine is determined by the coefficient of pressure increase during the isochoric delivery of heat λ_p .

$$p_z = \lambda_p p_c = \lambda_p p_a \varepsilon^\kappa. \quad (13)$$

For the generalised engine work cycle is:

$$q'_1 = c_v (T'_z - T_c) = c_v (\lambda_p T_c - T_c) = c_v T_c (\lambda_p - 1),$$

thus:

$$\lambda_p = \frac{q'_1}{c_v T_c} + 1. \quad (14)$$

Substituting relationship (14) to (13) we obtain:

$$p_z = p_a \cdot \varepsilon^\kappa \left(\frac{q'_1}{c_v T_c} + 1 \right) = p_a \varepsilon^\kappa \left(\frac{q'_1}{c_v T_a \varepsilon^{\kappa-1}} + 1 \right) = \frac{p_a \varepsilon q'_1}{c_v T_a} + p_a \varepsilon^\kappa. \quad (15)$$

Relationship (15) is true for all work cycles of piston engines. Analysis of that relationship shows that the value of maximum combustion pressure depends on the engine design – ε , the quality of fuel fed to the engine – κ , the engine supercharging or lack thereof – T_a , and the organization of the combustion process expressed by the quantity q'_1 , i.e. the quantity of fuel combusted in the cylinder according to the kinetic combustion mechanism.

5. Comparative working cycles of piston engines

Comparative work cycles of combustion engines permit an analysis of the phenomena occurring in the engine cylinder to their course in a real engine, which constitutes a better approximation than in the case of theoretical cycles.

The diagrams of comparative engine work cycles are made in p-V coordinates with the following assumptions:

- the working medium realising that cycle is an ideal, semi-ideal, or real gas,
- the mass of the working medium taking part in the work cycle is constant,
- the compression and expansion process proceeds in polytropic manner,
- heat is delivered to the work cycle in result of combustion occurring at constant volume, or at constant pressure, or at both constant volume and constant pressure, taking the incomplete and imperfect combustion into account,
- considered is the work of pump strokes going for realisation of the process of filling and exhaust of the exhaust gas from the cylinder, which are performed at a constant mean pressure in the cylinder.

As a rule, the work of pump strokes is negative for unsupercharged engines, and positive for supercharged engines.

Apart from the above mentioned assumptions, other assumptions could also be made that permit even better approximation of the prepared diagram of comparative work cycle to its effective cycle.

Example diagrams of comparative work cycles of an unsupercharged engine and a supercharged engine are presented in Fig. 3.

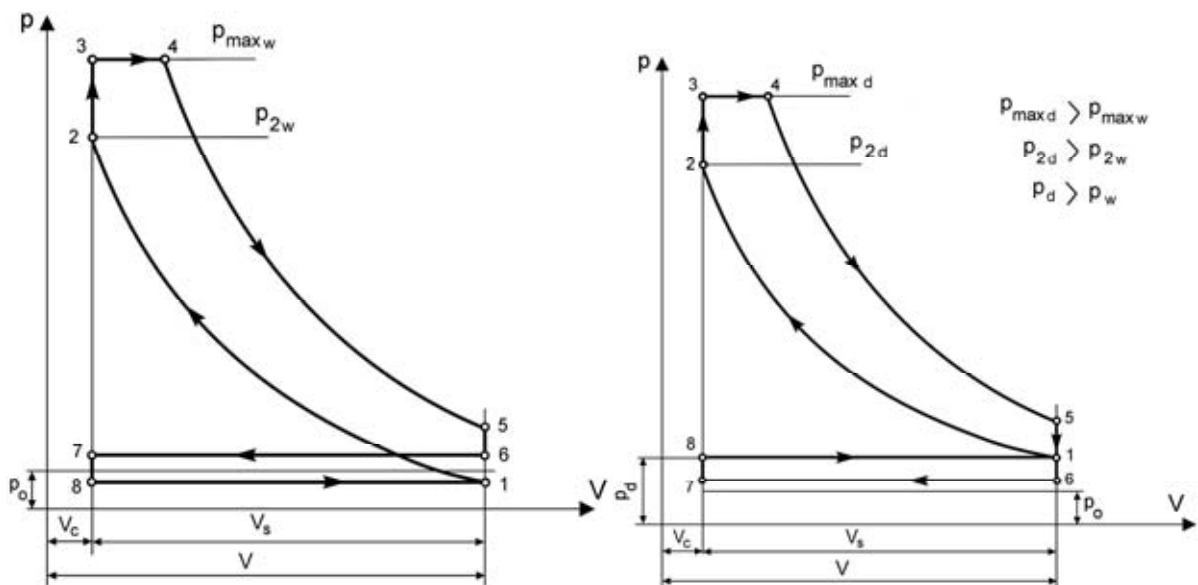


Fig. 3. Comparative work cycles of: a – an unsupercharged engine, b – a supercharged engine. The subscripts “w” and “d” refer to the unsupercharged engine and supercharged engine, respectively

6. Real working cycle of four-stroke piston engines

Effective indicator diagrams are made on the basis of measurement of the course of pressure change in the cylinder in function of the crank angle $\alpha \in \langle 0, 720^\circ \text{ Crankshaft Angle} \rangle$ and then we call them developed (open) indicator diagrams.

Using the dependency of the cylinder volume on the engine crank angle, the developed indicator diagram in $p-\alpha$ coordinates may be presented in $p-V$ coordinates. Examples of effective indicator diagrams for a four-stroke unsupercharged and supercharged engine are presented in $p-V$ coordinates in Fig. 4 and Fig. 5.

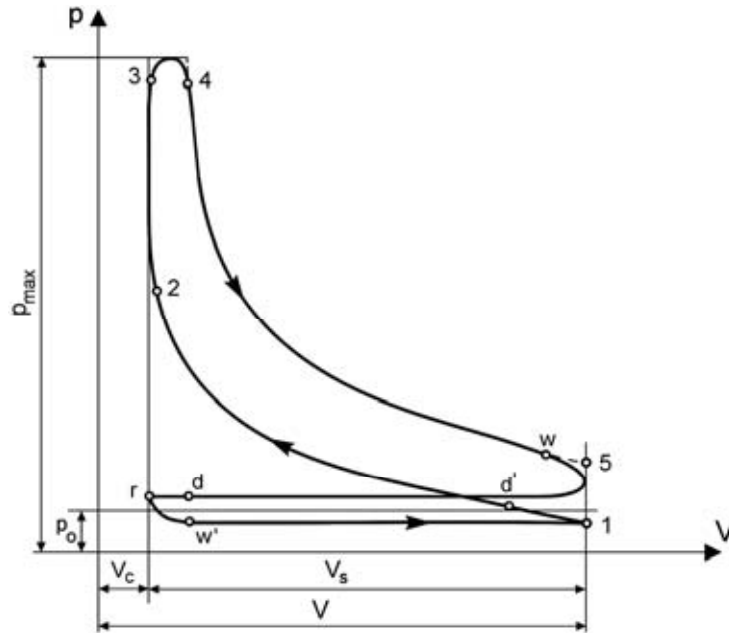


Fig. 4. The diagram for the effective work cycle of a four-stroke unsupercharged piston engine in $p-V$ coordinates, where d, d' – opening and closing, respectively, of the inlet valve; w, w' – opening and closing, respectively, of the outlet valve

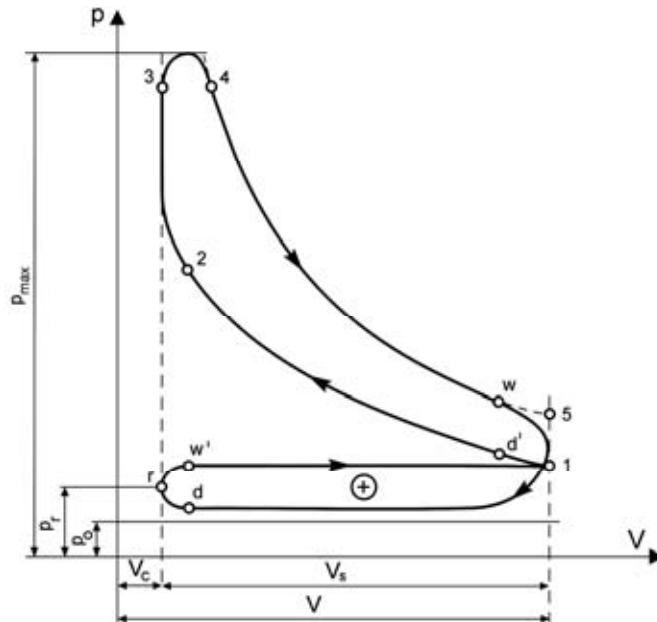


Fig. 5. The diagram for the effective work cycle of a four-stroke supercharged piston engine in $p-V$ coordinates, where d, d' – opening and closing, respectively, of the inlet valve; w, w' – opening and closing, respectively, of the outlet valve

The differences between the theoretical and effective engine work cycles result from the following causes:

- delivery and carrying away of heat does not take place by heating and cooling of the working medium, but as a result of combustion that may proceed with various speed and in a complete or incomplete, perfect or imperfect manner,
- apart from the isochoric and isobaric delivery of heat, taking place in the generalised theoretical thermodynamic work cycle, in the effective cycle there occurs a continuous exchange of heat between the gases and the cylinder and head walls, and further between the walls and the cooling medium,
- the process of filling the cylinder with a fresh load and removal of the exhaust gas from the cylinder is connected with the performing of work of pump strokes, connected with the resistance of flow in the inlet and outlet ducts and at the inlet and outlet valves,
- after the exhaust of the exhaust gas is completed, some quantity of the exhaust gas always remains in the cylinder, which means that after the process of filling the cylinder is completed, the cylinder is filled with a mixture of the fresh load and the exhaust gas,
- the values of specific heats of the working medium are variable and they depend on the temperature and its composition,
- hot walls separating the space occupied by the working medium cause the heating of the fresh load sucked in, which causes the reduction of the cylinder filling ratio,
- a change of the quantity of moles of the working medium in the cylinder occurs during the combustion process,
- the quantity of the working medium realising the work cycle is variable, which is caused by the losses of the medium escaping through cylinder leaks.

Having an effective indicator diagram of a combustion engine in p-V coordinates, one can describe on it a computational indicator diagram and, as is made in the theory of piston engines, use approximate methods for calculations of the processes creating the thermodynamic engine work cycle. The basis of those methods is constituted by: the volume balance equation, the working medium quantity balance equation, the energy conservation and change equation (the equation of the first principle of thermodynamics), and the state equation. Now let us consider a generalised mathematical model of the work process in the cylinder of a piston engine.

7. Conclusions

The theoretical heat efficiency of the generalized thermodynamical working cycle of combustion engine depends on design parameters of the engine brought out ε parameter, on fuel quality which the engine we is fed, (what expresses adiabatic exponent κ), and on the organization of combustion process determined with values of λ , p , ρ and δ . An efficient way of increasing of the average theoretical pressure value, so and engine power is increasing of pressure value of compressing beginning p_a . One of ways of value p_a increasing is application of supercharging for combustion engine. Comparative working cycles of combustion engines make possible more approximate analysis than theoretical cycle's phenomena occurring in the engine cylinder. Differences between theoretical and real working cycles of the engine are results of delivery and collect of heat takes place by the heating and cooling of the working charge, but in consequence of combustions which can run with the different rate and in the way complete or incomplete. Quantity of working charge realizing working cycle is variable, what is caused by losses of charge which leaks out.

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